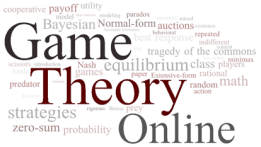

Strictly Dominated Strategies & Iterative Removal

Game Theory Course:
Jackson, Leyton-Brown & Shoham

"Rationality"

- A basic premise: players maximize their payoffs

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- ...

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Strictly Dominated strategies

- A strictly dominated strategy can never be a best reply
- Let us remove it as it will not be played
- All players know this - so let us iterate...
- Running this process to termination is called the iterated removal of strictly dominated strategies.

Strictly Dominated strategies

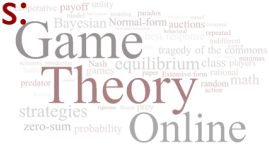
- A strategy $a_i \in A_i$ is strictly dominated by $a_i' \in A_i$ if

$$u_i(a_i, a_{-i}) < u_i(a_i', a_{-i}) \ \ \forall\, a_{-i} \in A_{-i}$$

Iterated Removal of Strictly Dominated Strategies: Example

	L	C	R
U	$3, 0$	$2, 1$	$0, 0$
M	$1, 1$	$1, 1$	$5, 0$
D	$0, 1$	$4, 2$	$0, 1$

Iterated Removal of Strictly Dominated Strategies: Example



	L	C	R
U	$3, 0$	$2, 1$	$0, 0$
M	$1, 1$	$1, 1$	$5, 0$
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- R is strictly dominated by C.

Iterated Removal of Strictly Dominated Strategies: Example

	L	C
U	$3, 0$	$2, 1$
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Iterated Removal of Strictly Dominated Strategies: Example

	L	C
U	$3, 0$	$2, 1$
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D	$0, 1$	$4, 2$

- M is strictly dominated by U.

Iterated Removal of Strictly Dominated Strategies: Example

	L	C
U	$3, 0$	$2, 1$
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Iterated Removal of Strictly Dominated Strategies: Example

	L	C
U	$3, 0$	$2, 1$
D	$0, 1$	$4, 2$

- L is strictly dominated by C.

Iterated Removal of Strictly Dominated Strategies: Example

	C
U	$2, 1$
D	$4, 2$

Iterated Removal of Strictly Dominated Strategies: Example

	C
U	$2, 1$
D	$4, 2$

- U is strictly dominated by D.

Iterated Removal of Strictly Dominated Strategies: Example

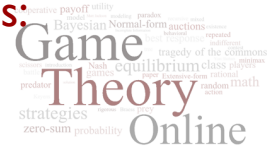
	L	C	R
U	$3, 0$	$2, 1$	$0, 0$
M	$1, 1$	$1, 1$	$5, 0$
D	$0, 1$	$4, 2$	$0, 1$

Iterated Removal of Strictly Dominated Strategies: Example

	L	C	R
U	$3, 0$	$2, 1$	$0, 0$
M	$1, 1$	$1, 1$	$5, 0$
D	$0, 1$	$4, 2$	$0, 1$

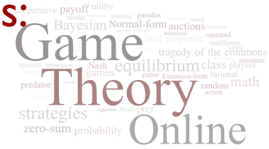
- A unique Nash equilibrium D, C.

Iterated Removal of Strictly Dominated Strategies: Another Example



	L	C	R
U	$3, 1$	$0, 1$	$0, 0$
M	$1, 1$	$1, 1$	$5, 0$
D	$0, 1$	$4, 1$	$0, 0$

Iterated Removal of Strictly Dominated Strategies: Another Example



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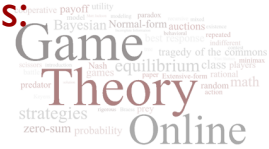
- R is dominated by L or C.

Iterated Removal of Strictly Dominated Strategies: Another Example



	L	C
U	$3,1$	$0,1$
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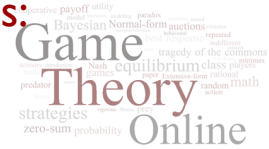

	L	C
U	$3,1$	$0,1$
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- M is dominated by the mixed strategy that selects U and D with equal probability.

Iterated Removal of Strictly Dominated Strategies: Another Example

	L	C
U	$3, 1$	$0, 1$
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- M is dominated by the mixed strategy that selects U and D with equal probability.
- Can use mixed strategies to define domination too!

Iterated Removal of Strictly Dominated Strategies: Another Example

	L	C
U	$3, 1$	$0, 1$
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Iterated Removal of Strictly Dominated Strategies: Another Example

	L	C
U	$3, 1$	$0, 1$
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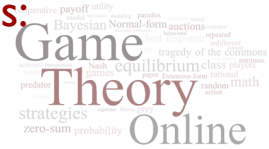
- No other strategies are strictly dominated.

Iterated Removal of Strictly Dominated Strategies: Another Example



	L	C
U	$3, 1$	$0, 1$
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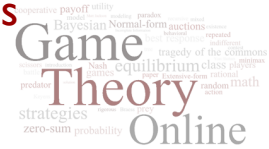
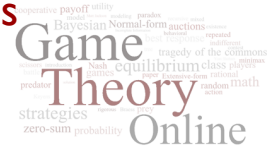
- No other strategies are strictly dominated.
- What are the Nash Equilibria?

Iterated Removal of Strictly Dominated Strategies



- This process preserves Nash equilibria.
  - It can be used as a preprocessing step before computing an equilibrium
  - Some games are solvable using this technique - those games are dominance solvable.

Iterated Removal of Strictly Dominated Strategies



- This process preserves Nash equilibria.
  - It can be used as a preprocessing step before computing an equilibrium
  - Some games are solvable using this technique - those games are dominance solvable.

- What about the order of removal when there are multiple strictly dominated strategies?
  - doesn't matter.

Weakly Dominated Strategies

- A strategy $a_i \in A_i$ is weakly dominated by $a_i' \in A_i$ if

$$u_i(a_i, a_{-i}) \leq u_i(a_i', a_{-i}) \text{ for all } a_{-i} \in A_{-i}, \text{ and}$$

$$u_i(a_i, a_{-i}) < u_i(a_i', a_{-i}) \text{ for some } a_{-i} \in A_{-i}.$$

- Can remove them iteratively too, but:

Weakly dominated strategies:

- They can be best replies.
- Order of removal can matter.
- At least one equilibrium preserved.
- Remember the Keynes Beauty Contest Game? Can you solve it via iterative elimination of Weakly Dominated Strategies?

Summary: Iterative Strict and Rationality

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Summary: Iterative Strict and Rationality

- Players maximize their payoffs
  - They don't play *strictly* dominated strategies
  - They don't play *strictly* dominated strategies, given what remains...
- Nash equilibria are a subset of what remains
- Do we see such behavior in reality?